

# Soret and Dufour Effects on MHD Free Convective Flow over a Permeable Stretching Surface with Chemical Reaction and Heat Source

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**Abstract:** In this chapter we consider a convective flow in a porous medium of an incompressible viscous fluid over a permeable stretching surface with chemical reaction and heat source. Using a similarity transformation the governing equations of the problem are reduced to first order linear differential equations. The governing equations are solved numerically by using shooting technique. The nonlinear differential equations for various values of the physical parameters are shown graphically.

**Index terms:** Radiation, Porous medium, Heat sink, Chemical reaction, Soret and Dufour effect.

## 1 INTRODUCTION

The study of viscous fluid flow due to stretching surface is an important type of flow occurring in several engineering process. Such process are wire drawing, heat-treated materials travelling between a feed roll and a wind-up roll or materials manufactured by extrusion, glass-fiber and paper production, cooling of metallic sheet or electronic chips, crystal growing, drawing of plastic sheets and many others. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching. Recently, Chiam [1] analyzed steady two-dimensional stagnation-point flow of an incompressible viscous fluid towards a stretching surface. Afify [2] have investigated similarity solution in MHD effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction/injection. Cortell [3] studied viscous flow and heat transfer over a nonlinearly stretching sheet. Dutta et.al [4] have discussed temperature field in floe over a stretching sheet with uniform heat flux. Nazar et.al [5] had studied stagnation point flow of a micro polar fluid towards a stretching sheet. Vidyasagar et.al [6] studied the effects of MHD flow over a permeable stretching surface with suction and internal heat generation/absorption. Krishna et.al [7] studied the effects of thermal radiation and chemical reaction on the steady two-dimensional stagnation point flow of a viscous incompressible electrically conducting fluid over a stretching surface with suction in the presence of heat generation. Ramana et.al [8] studied thermal diffusion and chemical reaction effects on unsteady MHD dusty viscous flow.

Vajravelu [9] studied viscous flow over a nonlinearly stretching sheet. The investigation of radiation and chemical reaction on MHD boundary layer flow over a moving vertical porous plate with heat in the presences of transverse magnetic field have been done by Krishna et.al [10]. The accompanying heat transfer problem has been studied more recently by Anderson et.al [11]. It is well known that the flow and heat transfer characteristics is affected significantly by the physical properties of the fluid, the study of non-Newtonian fluid flow and heat transfer over a stretching surface may gain importance.

Sandeep and Sugunamma [12] analyzed effects inclined magnetic field on unsteady free convective flow of a dissipative fluid past a vertical plate. Raju et.al [13] investigated radiation and soret effects on MHD nanofluid flow over a moving vertical plate in porous medium. Kafoussias and Williams [14] studied thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Recently, Anghel *et al.* [15] investigated the Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. Very recently, Postelnicu [16] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects.

In this chapter we analyze the Soret and Dufour effects on MHD free Convective Flow over a Permeable Stretching Surface with Chemical reaction and Heat sink by using the shooting technique. The present study is of immediate application to all those processes which are highly affected with heat enhancement concept. It is pertinent to mention here that some researchers have pursued their investigations with heat source but the effects of Soret and Dufour effect on MHD free Convective Flow over a Permeable Stretching Surface with Chemical reaction and Heat sink is presented in this chapter clearly.

## 2 MATHEMATICAL FORMULATIONS

We consider the steady two dimensional stagnation point flow of a viscous incompressible electrically conducting fluid near a stagnation point at a surface coinciding with the plane  $y = 0$ , with the flow being restricted to  $y > 0$ . Two equal and opposing forces are applied along the  $x$ -axis so that the surface is stretched (while keeping the origin fixed). The potential flow that arrives from the  $y$ -axis (impinges on the flat wall at  $y = 0$ ), divides into two streams on the wall and leaves in both directions. The flow is through a porous medium where the Darcy model is assumed. The viscous flow must adhere to the wall, whereas the potential flow slides along it. We denote the components of the fluid velocity by  $(u, v)$  at any point  $(x, y)$  for the viscous flow, while  $(U, V)$  denote the velocity components for the potential flow. We consider the case in which there may be a suction velocity  $(-W)$  on the stretching surface. Also, we denote the fluid temperature by  $T$ . The velocity distribution of the frictionless flow in the neighborhood of the stagnation point becomes

$$U(x) = ax, V(x) = -ay(1)$$

Where the parameter  $a > 0$  is proportional to the free stream velocity. The continuity and momentum equations for the two dimensional steady flow using the usual boundary layer approximation reduces to

**Continuity equation:**

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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

**Momentum equation:**

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + g\beta\rho(T - T_\infty) + g\beta^* \rho(C - C_\infty) - \sigma B_0^2 u - \frac{\mu}{k'} u \quad (3)$$

where

$$-\frac{\partial p}{\partial x} = U \frac{dU}{dx} + \frac{\sigma B_0^2}{\rho} U + \frac{v}{k'} U \quad (4)$$

**Energy equation:**

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) - \frac{\partial q_r}{\partial y} + v \left( \frac{\partial u}{\partial y} \right)^2 + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} \quad (5)$$

**Species diffusion equation:**

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} - K_l'(C - C_\infty) \quad (6)$$

On substituting the equation (4) in equation (3) we get

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = U \frac{dU}{dx} + \mu \frac{\partial^2 u}{\partial y^2} + g\beta\rho(T - T_\infty) + g\beta^* \rho(C - C_\infty) + \sigma B_0^2(U - u) + \frac{\mu}{k'}(U - u) \quad (7)$$

where  $u$  and  $v$  are the velocity components along the X and Y axes respectively,  $\mu$  is the coefficient of viscosity of the fluid,  $\nu$  is the kinematic viscosity,  $\beta, \beta^*$  are the thermal and concentration expansion coefficients respectively,  $\sigma$  is electrical conductivity,  $B_0$  is the uniform magnetic field,  $\rho$  is the density,  $T$  is the temperature inside the boundary layer,  $T_\infty$  is the temperature far away from the plate,  $C$  is the species concentration in the boundary layer,  $C_\infty$  is the species concentration of an ambient fluid,  $c_p$  is the specific heat at constant pressure,  $k$  is the thermal conductivity,  $k'$  is the porosity parameter,  $Q$  is the volumetric rate of heat generation,  $D$  is the molecular diffusivity of the species concentration and  $K_l'$  is the Chemical reaction parameter.

The corresponding boundary conditions are

$$u = cx, v = 0, T = T_w, C = C_w \text{ at } y = 0$$

$$u = ax, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (8)$$

where  $c > 0$ .

For an optically thin fluid, we have

$$\frac{\partial q_r}{\partial y} = 4\alpha^2 (T - T_\infty) \quad (9)$$

$$\text{Where } \alpha^2 = \int_0^\infty \delta \lambda \frac{\partial B}{\partial T} \quad (10)$$

Where  $\alpha^2$  is the absorption coefficient,  $\lambda$  is the frequency,  $B$  is the Planck's function and  $\delta$  is the radiation absorption coefficient.

In view of the equations (9) & (10), equation (5) reduces to

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) + \frac{16T_\infty^3 \sigma^*}{3k^*} \frac{\partial^2 T}{\partial y^2} \quad (11)$$

we introduce the following non-dimensional variables:

$$\eta = \sqrt{\frac{c}{\nu}} y, \quad \psi = \sqrt{c\nu} x f(\eta), \quad M = \frac{\sigma B_0^2}{\rho c},$$

$$H = \frac{Q}{c\rho c_p}, \quad \text{Pr} = \frac{\mu c_p}{k}, \quad K = \frac{\nu}{ck'},$$

$$Sc = \frac{\nu}{D}, \quad s = \frac{W}{\sqrt{c\nu}}, \quad Gr = \frac{g\beta(T_w - T_\infty)}{xc^2}, \quad (12)$$

$$K_l = \frac{K_l'}{c}, \quad Gc = \frac{g\beta^*(C_w - C_\infty)}{xc^2}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad R = \frac{4\alpha^2}{c\rho c_p}, \quad Sr = \frac{D_m K_T (T_w - T_\infty)}{T_w \nu (C_w - C_\infty)},$$

$$Ec = \frac{c^2 x^2}{c_p (T_w - T_\infty)}, \quad Du = \frac{D_m K_T (C_w - C_\infty)}{\rho \nu c_p c_s (T_w - T_\infty)}$$

where  $F(\eta)$  is a dimensionless stream function,  $\theta(\eta)$  is a dimensionless temperature of the fluid in the boundary layer region,  $\phi(\eta)$  is a dimensionless species concentration of the fluid in the boundary layer region and  $\eta$  is the similarity variable. The velocity components  $u$  and  $v$  are respectively obtained as follows

$$u(x, y) = \frac{\partial \psi}{\partial y} = xcf'(\eta), \quad (13)$$

$$v(x, y) = -\frac{\partial \psi}{\partial x} = -\sqrt{c\nu} f(\eta)$$

In view of the equations (12) and (13) the equations (6), (7) and (11) takes the form as below

$$f'''' + ff'' - (f')^2 + Gr\theta + Gc\phi + (M + K)(C - f') + C^2 = 0 \quad (14)$$

$$\theta'' + \text{Pr} Du\phi'' + \text{Pr} Ec(f'')^2 + \text{Pr}(f\theta' - f'\theta) + \text{Pr}(H - R)\theta = 0 \quad (15)$$

$$\phi'' + SrSc\theta'' + Sc(f\phi' - f'\phi) - K_l Sc\phi = 0 \quad (16)$$

Where the primes denote the differentiation with respect to  $\eta$ ,  $K$  is the Darcy permeability parameter,  $M$  is the magnetic parameter,  $C = a/c > 0$  is the stretching parameter,  $H$  is the heat source parameter,  $\text{Pr}$  is the Prandtl number,  $Sc$  is the Schmidt number,  $R$  is the Radiative parameter,  $Gr$  is the thermal Grashof number,  $Gc$  is the mass Grashof number and  $K_l$  is the dimensionless chemical reaction parameter,  $Sr$  is the Soret parameter,  $Du$  is the Dufour effect and  $Ec$  is the Eckert number.

The corresponding boundary conditions are

$$f' = 1, f = s, \theta = 1, \phi = 1 \text{ at } \eta = 0$$

$$f' = C, \theta = 0, \phi = 0 \text{ as } \eta \rightarrow \infty \quad (17)$$

### 3 SOLUTION OF THE PROBLEM

The governing boundary layer equations (14) to (16) corresponding to velocity, temperature and species concentration subject to the boundary conditions (17) are solved numerically by using shooting method. First of all higher order non-linear differential equations (14) to (16) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique.

### 4 RESULTS AND DISCUSSION

For numerical results we consider  $M = 1, \text{Pr} = 0.71, Du = 1, Sc = 0.60, Sr = 0.4, H = 0.1, K = 1, Ec = 0.001, Gr = Gc = 4, C = 0.5, K_l = 0.5$  and  $R = 0.5$ . These values kept as common in entire study except the varied values as displayed in respective figures..

Figures (1) - (3) represent the influence of the magnetic parameter  $M$  on the velocity, temperature and concentration profiles in the boundary layer, respectively. Application of a transverse magnetic

field to an electrically conducting fluid gives rise to a resistive-type force is called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer and to increase its temperature and concentration. Also, the effects on the flow and thermal fields become more so as the strength of the magnetic field increases.

The effects of Heat source parameter on velocity, temperature and concentration profiles are displayed in Figs. (4) - (6). It is observed that there is a increase in the velocity and temperature profiles with an increasing in  $H$ . This is due to the fact that when heat is generated. But, Figure 6 shows that the concentration decreases as  $H$  increases.

The influences of Radiation parameter on velocity, temperature and concentration profiles are displayed in Figs. (7) - (9). It is observed that an increase in  $R$  contributes to the decrease in the velocity and temperature distributions but it reverse inconcentration profiles.

Figures (10)-(12) display results for the velocity, temperature and concentration distribution, respectively. As shown, the velocity and temperature are increases with the increase of Dufour effect  $Du$ . But, Figure 12 shows that the concentration decreases as  $Du$  increases.

Figures(13)-(15) illustrate the influence of Soret parameter  $Sr$  on velocity, temperature and concentration profiles respectively. It is clear that there is an increase in the velocity and concentration profiles with an increasing in  $Sr$  but it reverse in temperature.

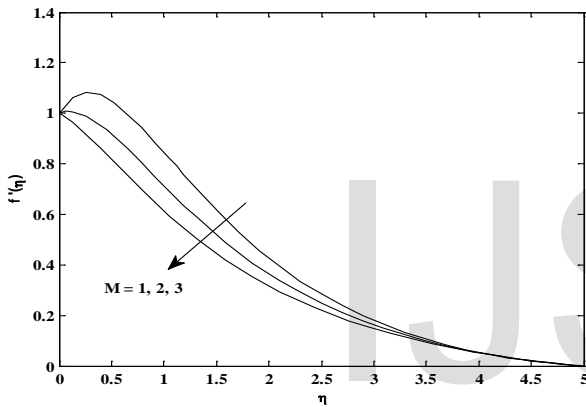


Fig.1: Velocity profiles for different values of  $M$

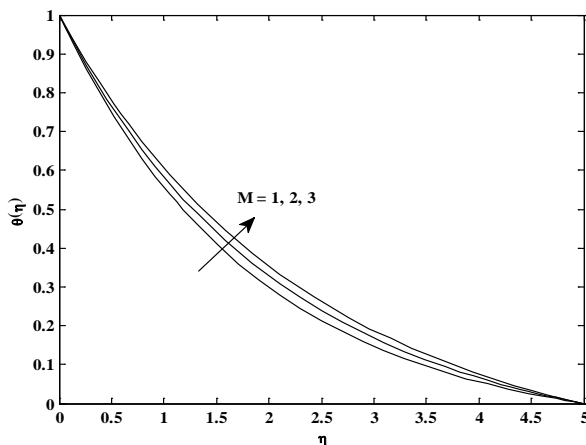


Fig.2: Temperature profiles for different values of  $M$

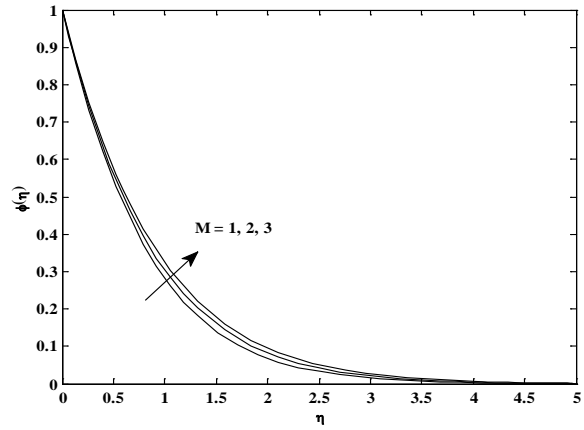


Fig.3: Concentration profiles for different values of  $M$

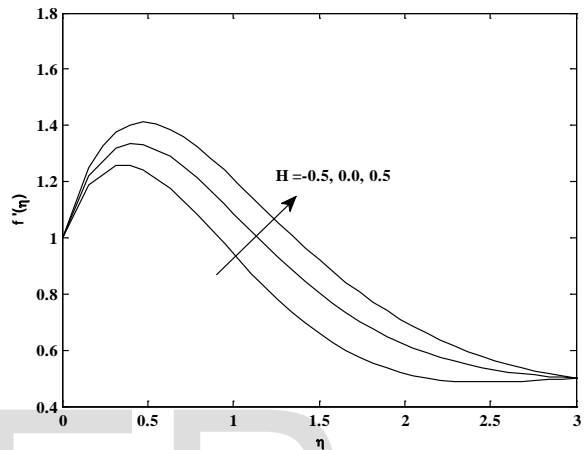


Fig.4: Velocity profiles for different values of  $H$

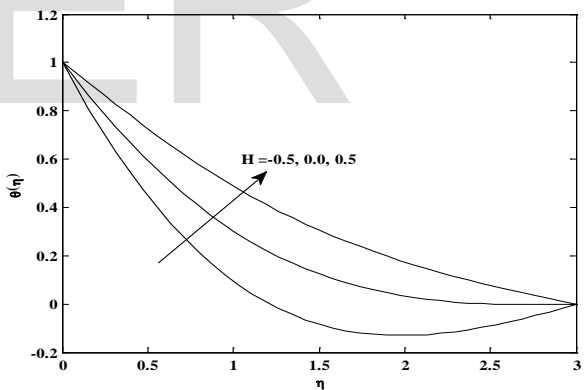


Fig.5: Temperature profiles for different values of  $H$

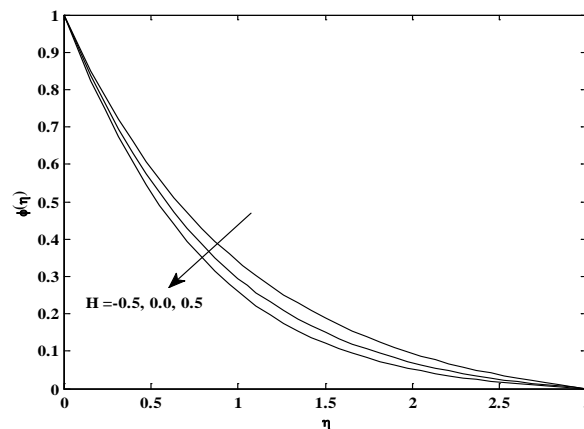


Fig.6: Concentration profiles for different values of  $H$

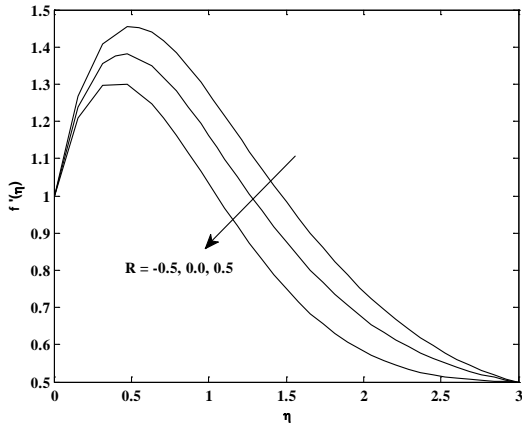


Fig.7: Velocity profiles for different values of R

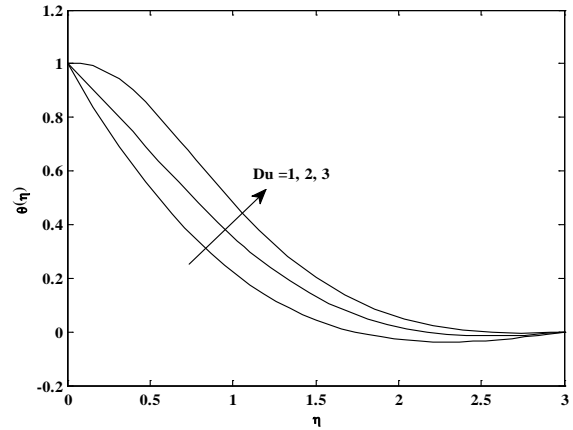


Fig.12: Temperature profiles for different values of Du

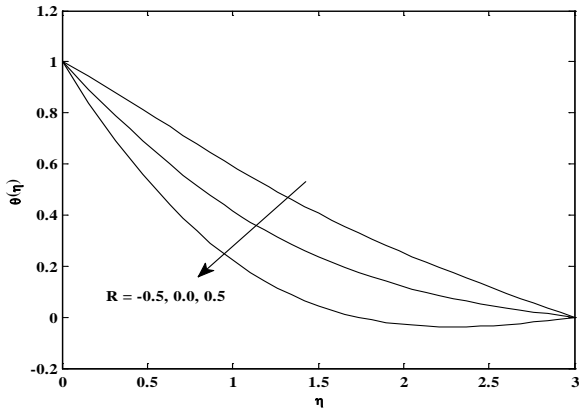


Fig.8: Temperature profiles for different values of R

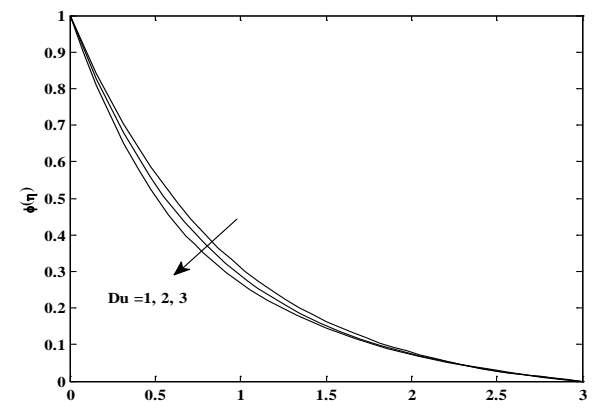


Fig.12: Concentration profiles for different values of Du

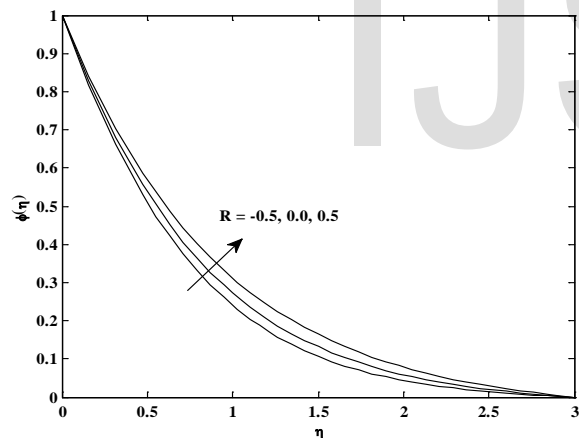


Fig.9: Concentration profiles for different values of R

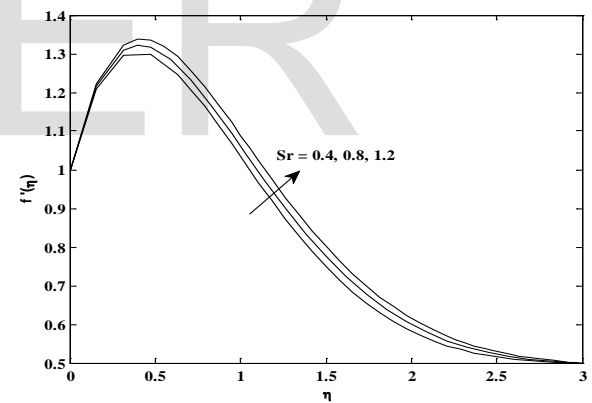


Fig.13: Velocity profiles for different values of Sr

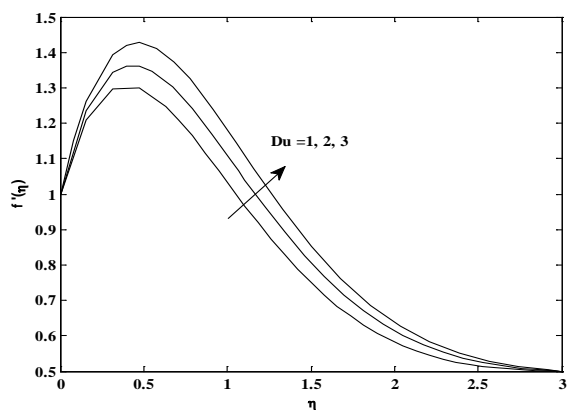


Fig.10: Velocity profiles for different values of Du

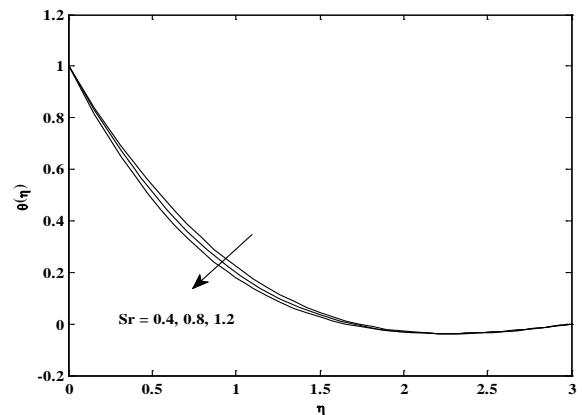


Fig.14: Temperature profiles for different values of Sr

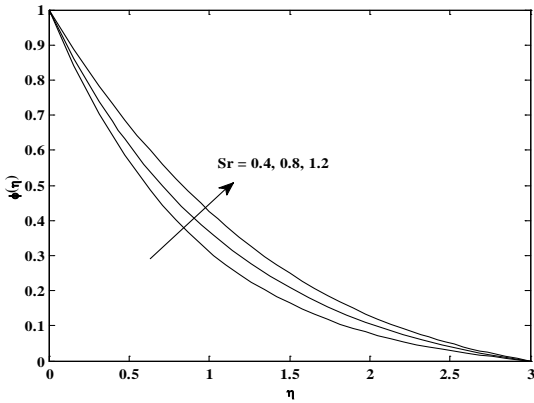


Fig.15: Concentration profiles for different values of Sr

#### 4 CONCLUSIONS

In this study we are investigated sores and dufour effect on the steady two dimensional stagnation point flow of a viscous incompressible electrically conducting fluid over a stretching surface with chemical reaction and heat source. The governing boundary layer equations are transformed to nonlinear ordinary differential equations by using similarity transformation which are then solved numerically. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs. The conclusions are made as follows:

- The velocity and concentration profile increases with the increase of sores parameter but it is decreases in temperature profile.
- The increase in heat source parameter increases the fluid temperature, which leads to an increase in rate of heat transfer.
- The velocity and temperature profile increases with the increase of dufour parameter but it is decreases inconcentration profile.

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